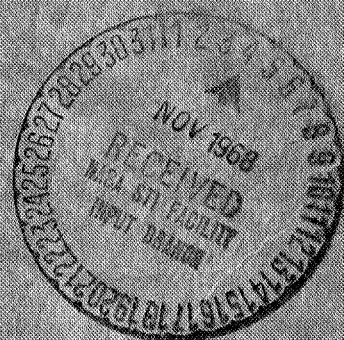
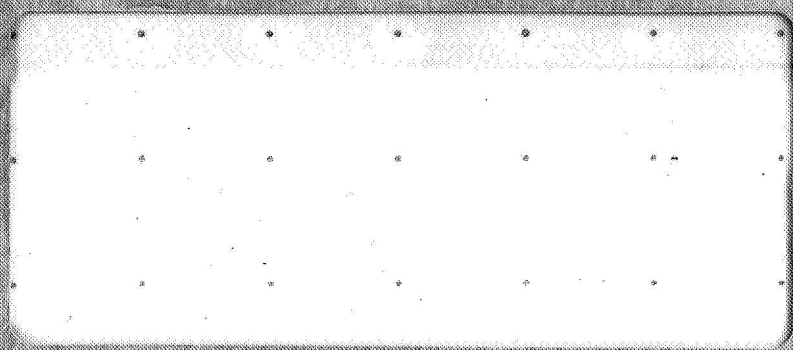


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**A NEW PRINCIPLE ON THE EXISTENCE OF  
HIGH-TEMPERATURE PLAZMOIDES**

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Translated by Joseph L. Zygielbaum

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## ABSTRACT

A detailed account of the effect of four factors on a cluster of charged particles of one sign "nuclei or electrons" establishes the presence of a determined stationary state of movement of particles in a cluster. The four factors considered are the following: an external magnetic field, a parallel axis of the cluster, the rotation of the cluster by a proper thrust, the forces of reciprocal repulsion of particles due to the presence of a circumferential charge and a temperature dispersion of velocities.

This stationary state is strongly characterized by an outlined cylindrical boundary of the cluster, the radius of which as it appears is involuntary and is determined by the aforementioned factors. Namely, there exists a determined correlation between the effective radius, the temperature, and the magnetic field of the cluster. From the respective values of magnetic fields and dimensions, one might anticipate temperatures on the order of  $10^{10}$  to  $10^{12}$ °K. The cluster rotates not cyclotronically but at least along a periphery with a Larmor frequency.

During determined conditions of concentration, the magnetic field of the clusters is also stable in relation to the changes of these magnitudes. A haphazard infraction of stability takes place (branching points appear) when the indicated conditions extend beyond their limits.

# I. INTRODUCTION

## Integral Equation of State. Examination of a Simple Model

A classical method for the description of static associations of particles is usually based on the separation of circumferential properties of media. This method is reflected by a determined normalization which is placed on the proposed solutions for static equations

$$N \rightarrow \infty; \quad V \rightarrow \infty; \quad \frac{N}{V} = \text{const.} < \infty$$

where  $N$  is the overall number of particles and  $V$  is the circumference of the system. This condition does not permit a reflection of the space-limited accumulation of particles which are supported by their own as well as external forces (without the participation of panels). In order to describe similar space-limited accumulations in one, two, or three measurements (which might be called plazmoides), it is necessary to introduce the following condition:

$$\iint_{(\infty)} f(r, v, t) dr dv < \infty \quad (1.1)$$

where  $f$  is the function of distribution of particles which determines the probable number of particles in an interval

$$dr dv = dx dy dz dv_x dv_y dv_z$$

In order to satisfy this condition, the function of propagation should decrease sufficiently fast in space coordinates while the velocities of these variables have an unlimited increase. A decrease with the increase of velocities is usually accomplished as a result of the Maxwell function of propagation. The necessary decrease in space coordinates from the center of the plazmoide permits one to present its effective dimensions as follows:

$$\int_{(\infty)} \rho dr = \rho_0 V_{eff}, \rho(r) = \int_{(\infty)} f dv$$

where  $\rho_0$  is the concentration of particles in the center of the plazmoide and  $V_{eff}$  is its effective circumference.

On the other hand, the function of propagation as a solution of the respective kinetic equation for a stationary condition should contain the following parameters of the system: temperature ( $\theta$ ), concentration ( $\rho_0$ ), magnetic field ( $H$ ), drift velocity ( $u$ ), etc. Thus, the condition (1.1) will become (Ref. 3):

$$\rho_0 V_{eff} = \phi(\theta, \rho_0, H, u, \dots) \quad (1.2)$$

This equation might be defined as an integral equation of the condition of a plazmoide. The value of this equation consists of the presence of a contact between the dimension of the plazmoide and the internal and external parameters, since this connection can be subjected to experimental research. This connection might be examined as a criterion of the stationary (quasi-stationary) state of the plazmoide. One of the important problems in the theory of plazmoides should consist of the establishment of a correlation (1.2) for concrete cases.

Of what type should the general character of such a connection be? This question can be answered by utilizing the ideas on dimensions. At least three characteristic natural frequencies of the system exist for the association of charged particles

$$\omega_0^2 = \frac{4\pi \rho_e^2}{\rho}; \quad \omega_j^2 = \frac{4\pi (j_e/c)^2}{\rho}; \quad \omega_L^2 = \left( \frac{eH}{2mc} \right)^2$$

where  $\rho_e$  and  $j_e$  are the densities of charge and electrical current,  $\rho$  is the mass density,  $\omega_0$  is the Langmuir frequency and  $\omega_L$  is the Larmor frequency.

If one should introduce two energy scales:  $\theta = kT$  and  $mc^2$  where  $m$  is the mass of the particle, then there are three characteristic lengths

$$l_0^2 = \frac{\theta}{m\omega_0^2}; \quad l_j^2 = \frac{\theta}{m\omega_j^2}; \quad l_L^2 = \frac{\theta}{m\omega_L^2} \quad (1.3)$$

and three additional lengths if one replaces  $\theta$  by the value  $mc^2$ .

The formulas (1.3) also belong to the type of correlations as (1.2). It is obvious that the known condition of the "ping" effect (Ref. 2;  $n\theta = H_s^2/8\pi$ , where  $H_s$  is the natural magnetic field) is connected with another of the correlations (1.3). In this paper a special view of an electromagnetic plazmoide which is related to the third correlation of the system (1.3) will be investigated.

For the purpose of visibility, it is convenient to begin investigations with a simple model so that it will be possible to compare and verify according to a strict theory the results which will be obtained later from a more complex model.

Assume that a continuously distributed circumferential charge of a cluster of atomic nuclei can be replaced by a superficial charge on a cylinder with a radius equal to the radius of the original cluster. In both cases, on one cylinder length unit should be located one and the same total charge. Assume that the external constant magnetic field  $H$  is directed along the cluster and that the cluster executes a left-hand thread rotation with an angular velocity of  $\omega$ . These two reasons lead to the appearance of a pressure directed inside of the cylinder, with a magnitude of  $(\sigma/c)(vH)$  where  $\sigma$  is the superficial density of the charge and  $v$  is the linear rotation velocity ( $v = \omega D$ , where  $D$  is the radius of the cylinder). For balance, the external pressure should be compensated for by the internal pressure which consists of the temperature pressure  $n\theta$  ( $n$  is the concentration of particles in the cluster,  $\theta = kT$ ), from the forces of electrostatic refraction  $1/2 \sigma E$  ( $E = 4\pi\sigma$ ), and the pressure which is stipulated by the centrifugal forces,  $(m/e)(\sigma v^2/D)$  where  $m$  and  $e$  are the mass and charge of a particle in the cluster.

$$n\theta + \frac{1}{2} \sigma E + \frac{m}{e} \frac{\sigma v^2}{D} = \frac{\sigma}{c} vH$$

From that, one finds the temperature value if it is taken into consideration that

$$\sigma = en \pi D^2 / 2\pi D = \frac{e}{2} nD$$

$$\theta = \frac{mD^2}{2} \left( 2\omega\omega_L - \omega^2 - \frac{1}{4}\omega_0^2 \right) \quad (1.4)$$

$$\omega_L = \frac{eH}{2mc}, \quad \omega_0^2 = \frac{4\pi ne^2}{m}$$

The condition for real equilibrium ( $\theta > 0$ ,  $\omega_0^2 \neq 0$ ) can be accomplished only by a sufficiently large magnetic field  $H$  and by a frequency  $\omega$  which is placed in a determined interval:

$$\omega_L = \sqrt{\omega_L^2 - \frac{1}{4}\omega_0^2} < \omega < \omega_L + \sqrt{\omega_L^2 - \frac{1}{4}\omega_0^2}$$

The maximum temperature value is obtained by a Larmor rotational frequency of  $\omega = \omega_L = eH/2mc$ :

$$\theta = \frac{m\omega_L^2 D^2}{2} \left( 1 - \frac{1}{4} \frac{\omega_0^2}{\omega_L^2} \right) \quad (1.5)$$

This formula points out the presence of a basic property of the plazmoide: that its dimensions are not arbitrary but are determined by internal ( $\theta$ ,  $\omega_0^2$ ) and external ( $\omega_L$ ) conditions. The formula (1.5) makes more definite the condition (1.2).

Since the electrostatic refraction was entered only as a correcting factor (by sufficiently large  $H$ :  $(1/4)(\omega_0^2/\omega_L^2) < 1$ ), it is clear that the magnetic interaction of currents due to rotation can be disregarded because it is always less than the electrostatic forces on the factor  $v^2/c^2 < 1$ . For that same reason, the drive speed of the nuclei along the cluster did not enter into the formula (1.5). In the table are shown evaluations of the order of the appearing magnitudes  $T$ ,  $\nu = \omega/2\pi$ , and  $n$  for the deuterium in the case of certain values of  $H$  and  $D$ .

This evaluation shows that it is possible to reach high temperatures without the consumption of great electrical currents and represents the basic advantage of the demonstrated method.

| $H$<br>(Gauss) | $D$<br>(cm)    | $n$<br>(1/cm <sup>2</sup> ) | $\nu$<br>(1/sec) | $T$<br>(°K)       |
|----------------|----------------|-----------------------------|------------------|-------------------|
| $10^8$         | 10             | $< 2 \cdot 10^8$            | $3 \cdot 10^8$   | $2 \cdot 10^6$    |
| $10^4$         | $10^2$         | $< 2 \cdot 10^{10}$         | $3 \cdot 10^4$   | $2 \cdot 10^{10}$ |
| $5 \cdot 10^4$ | $2 \cdot 10^2$ | $< 10^{12}$                 | $10^2$           | $2 \cdot 10^{12}$ |

The correlation (1.5) indicates that it would have been possible also to study a plazmoide experimentally. First of all, it would have been possible to check the jump-like origin of a plazmoide state with an increase of  $H$ : when

$$\frac{1}{4} \frac{\omega_0^2}{\omega_L^2} < 1, \quad \theta < 0.$$

The second important property of a plazmoide is the magnitude of its natural rotational frequency. This frequency might belong to the Larmor type ( $\omega_L$ ), but not of the cyclotron type ( $2\omega_L$ ). The latter one only occurs in three cases when the cluster is not in a plazmoidal state. Namely, if the thermal dispersions in velocities are fully disregarded and also the activity of the circumferential charges (that is, if it is assumed that  $\theta = 0$ ;  $\omega_0^2 = 0$ ), then from the formula (1.2):

$$2\omega\omega_L = \omega^2$$

a cyclotron frequency appears.

Further, the temperature dependence on the successive number  $Z$  and the atomic weight of  $A$  nuclei and also on  $H$  and  $D$

$$\theta \sim \frac{Z^2 H^2 D^2}{A}$$

could have been the subjects of an experimental analysis. Of course, most important is the experimental establishment of the fact that the given type of plazmoide actually exists.

The theory has to serve two purposes. First, in which regard of the investigation will the ideal model of a pipe with a semicharged surface reflect a real cluster of atomic nuclei? Second, what kind of actual frequency of rotation should there be? And does there appear a similar frequency of rotation by itself due to the effect of the external magnetic field with a magnitude of  $H$ ?

## II. ORIGINAL EQUATION AND BOUNDARY CONDITIONS

### Propagation Function of Charged Particles in Internal Magnetic Field During Rotation

Assume that the following equation for the function of propagation of  $f$  (Ref. 1) is the original equation of the problem:

$$\left. \begin{aligned} \frac{\partial f}{\partial t} + \text{div}_{\mathbf{r}} \mathbf{v} f + \text{div}_{\mathbf{v}} \mathbf{g} f &= 0 \\ \mathbf{g} &= -\frac{e}{m} \nabla V + \frac{e}{mc} [\mathbf{v} \mathbf{H}] \\ \Delta V &= -4\pi e \int_{(\infty)} f d\mathbf{v} \end{aligned} \right\} \quad (2.1)$$

The boundary and supplementary conditions are the following: First, the conditions of cylindrical symmetry, namely, along the axis of the cylinder:

$$V(0) = \left. \frac{\partial V}{\partial r} \right|_{r=0} = 0 \quad (2.2)$$

Second, the condition of a plasmoidal state (1.1):

$$\int_0^\infty \left( \int_{(\infty)} f d\mathbf{v} \right) 2\pi r dr = \rho_0 \pi D^2 < \infty \quad (2.3)$$

where  $D$  is the effective radius of the cylinder. Finally, for the condition when  $H \rightarrow 0$  and  $\omega \rightarrow 0$ , the solution should be converted into a Maxwell function of distribution by velocities with a given thrust speed " $u$ " along the axis of the cluster.

In the cylindrical coordinates,  $r, \theta, z; v_r, v_\theta, v_z$ :

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r} \frac{\partial f}{\partial \theta} + v_z \frac{\partial f}{\partial z} + \left[ -\frac{e}{m} \frac{\partial V}{\partial r} + \left( v_\theta + \frac{e}{mc} Hr \right) \frac{v_\theta}{r} \right] \frac{\partial f}{\partial v_r} + \left( -\frac{e}{m} \frac{\partial V}{\partial z} \right) \frac{\partial f}{\partial v_z} - \left( v_\theta + \frac{e}{m} Hr \right) \frac{v_r}{r} \frac{\partial f}{\partial v_\theta} = 0 \quad (2.4)$$

In spite of the absence of a stochastic mechanism, the original equation has a solution in the form of a Maxwell function of distribution if a solution in the form of a function with separating variables is sought:

$$f = \rho(r, z - ut) w(v_r^2) w(v_\theta^2) w[(v_z - u)^2]$$

In the case of  $\rho$  and  $w$ , one finds

$$\rho = \exp \left( -\frac{eV}{\theta} \right), \quad w = \left( \frac{m}{2\pi\theta} \right)^{1/2} \exp \left( -\frac{mv^2}{2\theta} \right)$$

Judging from the superposed condition, one should search for the solution in the form most approximating the Maxwell distribution. In order that the solution contain the rotation of the system, it is necessary to neglect multiplicity. Since the multiple  $v_\theta r$  appears during the rotation because of the factor  $(v_\theta - u_\theta)^2$  where  $u_\theta = \omega r$ , this would indicate the absence of separation for the variables by  $r$  and  $v_\theta$ . It is assumed that

$$f = \rho(r, z - u t) w(v_r^2) w(v_\theta^2) w[(v_z - u)^2] \times F(r, v_\theta) \quad (2.5)$$

where  $F$  is applicable to the determination of the function, and considers the retreat from multiplicity. By substituting (2.5) into (2.4), all terms which do not contain derivatives reduce each other since the Maxwell distribution is a solution and will remain

$$v_r f_M \frac{\partial F}{\partial r} - v_r \frac{1}{r} \left( v_\theta + \frac{eH}{mc} r \right) \frac{\partial F}{\partial v_\theta} f_M = 0$$

where  $f_M$  identifies the function of distribution when  $F = 1$ , or

$$r \frac{\partial F}{\partial r} = \left( v_\theta + \frac{eH}{mc} r \right) \frac{\partial F}{\partial v_\theta}$$

This equation for  $F$  has a solution in the form of the arbitrary function  $\phi$ , which in a special way depends on  $r$  and  $v_\theta$ :

$$F(r, v_\theta) = \phi \left[ r \left( v_\theta + \frac{eH}{2mc} r \right) \right]$$

Now attention is turned to the appearance of "1/2" which differs from the preceding formulas. In what form will the function  $\phi$  be realized? In conformity with the Maxwell distribution, it should be considered in the form of an exponential function:

$$F(r, v_\theta') = e^{-Q}; \quad Q = \left( \frac{m\omega}{2\theta} \right) 2r v_\theta' - Ar^2 v_\theta'^2, \quad (2.6)$$

where  $Q$  is the polynomial of the original argument  $rv'_\theta$  and  $v'_\theta$  indicates the expression  $v_\theta + (eH/2mc)r$ . Since this case approaches most closely the function of a Maxwell distribution by velocities which also happens to be a polynomial of the second degree, only a polynomial up to the second order inclusive has been considered. The constants  $\omega$  and  $A$  are in mean time and are entirely arbitrary constants, the sense of which should be determined later on.

First to be examined are the results when only the first term in the expression  $Q$  is considered. The following equation

$$w(v_\theta^2) F(rv'_\theta) = \left( \frac{m}{2\pi\theta} \right)^{1/2} \exp \left[ -\frac{m}{2\theta} (v_\theta - u_\theta)^2 + S \right] \quad (2.7)$$

has a distribution function with a drop  $u_\theta$  and a potential function  $S$ . By comparing the above exponent with

$$-\frac{mv_\theta^2}{2\theta} - Q$$

one obtains the following expressions for  $u_\theta$  and  $S$ :

$$\left. \begin{aligned} u_\theta &= \omega_r \\ S &= \frac{m}{2\theta} (\omega^2 + 2\omega\omega_L) r^2 \end{aligned} \right\} \quad (2.8)$$

( $\omega_L = eH/2mc$ ); but when  $H = 0$ , it transfers to the known result on corrections to the Maxwell distribution due to the rotation of the system. The constant  $\omega$  is, in effect, a cyclic frequency of rotation.  $S = m\omega^2 (r^2/2\theta)$  is the "potential function" of the centrifugal force. When  $H \neq 0$ , there appears a new interference term which is stipulated for by the connection of rotation with the magnetic field. In the case of a left-hand rotation (when  $e > 0$ ):  $\omega < 0$  and by a sufficiently small rotational frequency

$$|\omega| < 2\omega_L = \frac{eH}{mc}$$

the factor  $\omega^2 + 2\omega\omega_L$  becomes negative. In the case of a Larmor rotational frequency ( $\omega = -\omega_L$ ):

$$S = - \frac{m\omega_L^2}{2\theta} r^2 \quad (2.9)$$

Under the effect of the magnetic field, the transition "of a potential function"  $S$  from a positive to a negative value is necessary for the appearance of a plazmoidal state since with that it is possible to anticipate the similarity of the integral (1. 1):

$$\int_0^\infty \frac{e^{-\frac{eV}{\theta}}}{r} dr < \infty$$

Thus the consideration of the first term in  $Q$  leads to a result which approximates the case of the elementary examination in Section I.

The rotational frequency  $\omega$  remains an arbitrary magnitude in this case also and leads to the necessity of investigating the second term of the polynomial. Here will be presented also an equation analogous with (2.7)

$$w(v_\theta) F(rv'_\theta) = \left( \frac{m}{2\pi\theta} \right)^{1/2} \exp \left[ - \left( \frac{m}{2\theta} + \alpha \right) (v_\theta - u_\theta)^2 + S \right]$$

where the values  $\alpha$ ,  $u_\theta$ , and  $S$  are determined by a comparison with (2.6). One obtains  $\alpha = Ar^2$  and

$$\left. \begin{aligned} u_\theta &= - \frac{\left( -\frac{m}{2\theta} \omega + A\omega_L r^2 \right) r}{\frac{m}{2\theta} + Ar^2} \\ S &= \frac{\left( \frac{m}{2\theta} \right)^2 \left( \omega^2 + 2\omega\omega_L - Ar^2 \frac{2\theta}{m} \omega_L \right) r^2}{\frac{m}{2\theta} + Ar^2} \end{aligned} \right\} \quad (2.10)$$

When  $H = 0$  but  $A \neq 0$ , the formulas (2.10) are converted into the Oorta formulas (Ref. 3) which are applied in the astral statistic

$$\left. \begin{aligned} u_{\theta} &= \frac{m}{2\theta} \frac{\omega r}{\frac{m}{2\theta} + Ar^2} \\ S &= \left( \frac{m}{2\theta} \right)^2 \frac{\omega^2 r^2}{\frac{m}{2\theta} + Ar^2} \end{aligned} \right\} \quad (2.11)$$

These formulas reflect the rotation of a galaxy  $G$ , which differs from the rotation of a solid body. When  $r \rightarrow 0$ ,

$$\begin{aligned} u_{\theta} &\rightarrow \omega r \\ S &\rightarrow \frac{m\omega^2 r^2}{2\theta} \end{aligned}$$

one deals with a common rotation; however, when  $r \rightarrow \infty$ ,

$$\begin{aligned} u_{\theta} &\rightarrow \frac{m}{2\theta} \frac{\omega}{A} \frac{1}{r} \rightarrow 0 \\ S &\rightarrow \left( \frac{m}{2\theta} \right)^2 \frac{\omega^2}{A} = \text{const.} \end{aligned}$$

the boundary region remains intact. Also in this case, the rotational frequency  $\omega$  remains arbitrary. The constant " $A$ " characterizes the appearing phenomena of "twisting" and also of "the second temperature" since the function of distribution takes on the following form

$$f = \rho \frac{1}{\pi^{3/2}} \sqrt{D} e^{-\frac{m}{2\theta} v_r^2 - \left( \frac{m}{2\theta} + Ar^2 \right) \left( v_{\pi} - u_{\theta} \right)^2 - \frac{m}{2\theta} \left( v_2 - u \right)^2}$$

where

$$\sqrt{D} = \frac{m}{2\theta} \sqrt{\frac{m}{2\theta} + Ar^2}$$

$$\rho = \rho(r, z - ut) = \int_{(\infty)} f dv$$

where  $u\theta$  and  $\rho$  are determined by formulas (2.10).

When  $\omega_L \neq 0$  and  $A \neq 0$ , two effects appear: the interference of the magnetic field with rotation and the interference of the magnetic field with the "appearance of twisting", and the appearance of the terms  $\omega\omega_L$  and  $Ar^2\omega_L$  (and also  $Ar^2\omega_L^2 (2\theta/m)$ ). It should also be noted that when  $r \rightarrow \infty$  the following takes place:

$$\left. \begin{aligned} u_\theta &\rightarrow -\omega_L r \\ S &= -\frac{m\omega_L^2 r^2}{2\theta} \end{aligned} \right\} \quad (2.12)$$

The first formula indicates that the rotation at the boundaries of this system is similar to the rotation of a solid body, whereby the direction and frequency of rotation coincides with the Larmor frequencies by their values. "The potential function  $S$ " has a sign which is contrary to the random rotation without a magnetic field. This secures the localization of the cluster with sufficiently outlined boundaries which is obvious from the appearance of the "cutoff" factor:

$$f \sim e^{-\frac{m\omega_L^2 r^2}{2\theta}}$$

Thereby it should be mentioned that the magnitude of the frequency  $\omega$  and the direction of rotation when  $r \rightarrow 0$  remains in theory arbitrary. In a specific case, by the values  $\omega = -\omega_L$  the formulas (2.12) are applicable for arbitrary  $r$ , which can be seen from the correlation (2.10).

### III. NONLINEAR EQUATION FOR DISTRIBUTION OF THE POTENTIAL AND ITS SOLUTION

By substituting the function of distribution which has been found in Section II in the second of the equations of the system (2.1), one will find

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dV}{dr} \right) = -4\pi e \rho_0 e^{-\frac{m \omega_L^2 r^2}{2\theta} - \frac{eV}{\theta}} \quad (3.1)$$

Introducing the dimensionless values

$$\frac{m \omega_L^2}{2\theta} r^2 = x^2; \quad -\frac{eV}{\theta} = \phi$$

one will put down the condition for the plasmoidal solution

$$\rho_0 \int_0^\infty e^{-\frac{m \omega_L^2 r^2}{2\theta} - \frac{eV}{\theta}} 2\pi r dr = \rho_0 \pi D^2 < \infty$$

in the form:

$$\frac{4\theta}{m \omega_L^2} \int_0^\infty x e^{-x^2 + \phi} dx = D^2 \quad (3.2)$$

The magnitude  $\theta$  with which one is interested will possibly be expressed as follows:

$$\theta = \frac{m \omega_L^2 D^2}{4} \frac{1}{\psi(\lambda)} \quad (3.3)$$

where  $\psi(\lambda)$  is the special function which is determined by the integral

$$\psi(\lambda) = \int_0^{\infty} x e^{-x^2 + \phi(x, \lambda)} dx \quad (3.4)$$

and, in its turn,  $\phi(x, \lambda)$  is the solution for Equation (3.1) with the respective conditions when  $x = 0$ :

$$\left. \begin{aligned} \frac{1}{x} \frac{d}{dx} \left( x \frac{d\phi}{dx} \right) &= \lambda e^{-x^2 + \phi} \\ \phi(0) &= \phi'(0) = 0, \end{aligned} \right\} \quad (3.5)$$

where the following is defined:

$$\lambda = 2 \frac{\omega_0^2}{\omega_L^2}$$

where  $\omega_0$  is the Langmuir frequency. Thus the problem of determining the integral equation of the condition (3.2) is reduced to a solution of Equation (3.4) with a successive determination of the form of function  $\psi(\lambda)$ .

A specific solution (3.4), by a fixed value of  $\lambda$ , is the solution  $\phi \equiv 0$  when  $\lambda = 0$ . The original conditions and equation are thereby satisfied. Further,

$$\psi(0) = \int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}$$

and consequently, Formula (3.2) has a simple form:

$$\theta = \frac{m \omega_L^2 D^2}{2} \quad (3.6)$$

which coincides accurately with the result of the elementary investigation (1.5), if one places there also  $\omega_0^2 = 0$ .

In order to determine the solution when  $x \rightarrow 0$  but  $\lambda \neq 0$ , an examination will be made of the equation for  $(d\phi/dx)$ , which is obtained by integration of (3.4) at the sector 0,  $\pi$

$$\frac{d\phi}{dx} = -\frac{\lambda}{x} \int_0^x x e^{-x^2 + \phi} dx$$

Since on the basis of the initial conditions when  $x \rightarrow 0$ ,  $\phi(x \rightarrow 0) \rightarrow 0$ , then

$$\frac{d\phi}{dx} \rightarrow -\frac{\lambda}{x} \int_0^x x e^{-x^2} dx \rightarrow -\frac{\lambda x}{2}$$

and consequently,

$$\phi(x) = \frac{\lambda x^2}{4} \quad (3.7)$$

This indicates that both initial conditions, which are not contrary to the equation, are fulfilled by an arbitrary value  $\lambda$ .

In order to determine  $\phi(x)$  when  $x \rightarrow \infty$ , it is convenient to transfer to the integral form of the equation. The integral presentation of a two-dimensional equation of Poisson

$$\Delta\phi = f(x, y)$$

has the following form in the region of infinity:

$$\phi(P) = -\frac{1}{2\pi} \iint_{(\infty)} f(A) \ln \frac{1}{r_{AP}} dS_A$$

If  $P \rightarrow \infty$ , then

$$\phi(P) \rightarrow -\frac{1}{2\pi} \ln \frac{1}{r} \iint_{(\infty)} f(A) dS_A$$

In dimensionless units when  $x \rightarrow \infty$ ,

$$\phi(x) \rightarrow \frac{m \omega_0^2 D^2}{2\theta} \ln x \quad (3.8)$$

if one takes advantage of (3.5) and (3.2). This result shows that the condition for a plazmoidal state is fulfilled

$$\int_0^{\infty} e^{-x^2 + a \ln x} x dx < \infty, \quad a = \frac{m \omega_0^2 D^2}{2\theta}$$

since the gradual increase of the subintegral function  $x e^{a \ln x}$  is eliminated by the cutoff factor  $\exp(-x^2)$ .

It is important to establish an analytical extension of the solution ( $\phi = 0, \lambda = 0$ ) in the area of sufficiently small  $\lambda$ , that is, to prove the existence of a solution at small  $\lambda$  and if possible its uniqueness. This may be accomplished based on the method of the long accomplished mathematical work of Bierberbach on the equation  $\Delta u = \lambda e^u$  (Ref. 4). Assume that  $w(x, y)$  is the solution to the Poisson equation  $\Delta w = 4\pi\rho$  ( $\rho > 0$ ), which corresponds in infinity with the solution of the equation

$$\Delta \phi = \lambda e^{-r^2 + \phi} \quad (r^2 = x^2 + y^2)$$

that is, with (3.8). From this is created the differential  $\phi - w = u$  which satisfies the equation:

$$\Delta u = \lambda e^{-r^2 + w + u} - 4\pi\rho$$

or

$$\Delta u - \lambda e^{-r^2 + w} u = \lambda e^{-r^2 + w^2} (e^w - u) - 4\pi\rho \quad (3.9)$$

It now becomes necessary to find the solution to (3.9) with the boundary conditions

$$u = 0, \quad r \rightarrow \infty$$

This problem is equivalent to the following integral equation:

$$u(x, y) = -\lambda \iint_{(\infty)} K(x, y; \xi, \eta) e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} (e^u - u) d\xi d\eta + \iint_{(\infty)} K 4\pi\rho d\xi d\eta \quad (3.10)$$

where  $K$  is the function of the Grin equation  $\Delta u - \lambda e^{-r^2 + w^2} u = 0$ .

The method of consecutive approximations in the following form will be applied:

$$u_0 = 0$$

$$u_{z+1} = -\lambda \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} (e^{u_z} - u_n) d\xi d\eta + \iint_{(\infty)} K 4\pi\rho d\xi d\eta \quad (3.11)$$

It was proven in the approximation 1, that a difference between the neighboring terms of the sequence satisfied inequality:

$$0 < u_n - u_{n+1} \leq S_0^{n+1} \quad (3.12)$$

where

$$S_0 = \max. \lambda \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} d\xi d\eta$$

From this equation, it is obvious that with a sufficiently small  $\lambda$  it is possible to prove that  $S_0 < 1$  and, consequently, to obtain a uniformly coinciding sequence. Thus the existence of a solution by a small  $\lambda \neq 0$  was proven. The uniqueness of the solution can be proven by utilizing the general methods for the solution of non-linear integral equations. (See Appendix II.)

Now it will be proved that by sufficiently large values of  $\lambda$  the uniqueness of the solution does not take place anymore, since a point of branching off of the equation appears. In fact, the initial equation (3.5) has a specific, but accurate solution  $\phi = x^2$  when  $\lambda = 4$ , which can be easily checked by substitution.

It is important from a physical viewpoint that this solution should not belong to the class of plasmoidal solutions, since in that case

$$\int_0^\infty x e^{-x^2 + \phi} dx = \infty$$

In that manner, when in the area of small values of  $\lambda$  (that is, by sufficiently large magnetic fields) a single plasmoidal state takes place, then in the case of small fields of  $H$  (when  $\lambda \geq 4$ ) nature should make a choice between a plasmoidal and nonplasmoidal state. Roughly, this corresponds to the elementary examination in Section I. In that examination  $1/4 (\omega_0^2/\omega_L^2) < 1$  was also required for the existence of the plazmoide.

In this specific case where  $H = 0$ , the original equation (3.1) has the following form

$$\frac{d^2\phi}{dr^2} + \frac{1}{2} \frac{d\phi}{dr} = \lambda e^\phi; \quad \lambda = \frac{4\pi e^2 \rho_0}{\theta}$$

In Appendix III it is shown that the solution of this equation can be achieved by the stipulation

$$\phi(0) = 0 \quad \phi(D) = \infty$$

In this case the concentration distribution has the following form

$$\rho = \rho_0 \frac{1}{\left[1 - \frac{r^2}{D^2}\right]^2}$$

This solution is not a plasmoidal solution; it is singular and requires for its realization impermeable panels when  $r = D$ .

#### IV. SUMMARY

1. In the case of a space-limited plasma, which consists of even one type of charged particles contained by internal (and external) forces within the walls (in the case of plazmoides), a so-called integral equation of state should exist, that is, a fully determined correlation which connects the linear dimensions of the plazmoide with its internal characteristics: temperature, concentration, magnetic field, etc.

2. In the presence of an external magnetic field, a cylindrical cluster of atomic nuclei with a drift velocity along the field might exist under the conditions of a plazmoidal state; that is, there is in existence such a stationary state of particle movement in the cluster, when its cross dimensions are not arbitrary but are determined by its internal and external conditions. A plazmoide has proven to be a result of the presence of three factors: the external magnetic field, the natural mechanical rotation, and the temperature distribution of velocities. The distribution function of particles in a magnetic field in the presence of rotation was determined. It was shown that the rotational velocity depends nonlinearly on the radius of the cylindrical cluster. On the other hand, at the periphery the cluster rotates as a solid body with a frequency and direction which coincide accurately with the Larmor values and not with cyclotronic values. Under known conditions the found function of distribution satisfies the criterion of existence of a plazmoidal state.

3. A similar plazmoide is characterized by a sufficiently strong cylindrical outline and by a determined effective radius which depends on the temperature, the magnetic field, and the concentration of particles. The individualization of atomic nuclei was included in the integral equation of state through the proper number and atomic weight. A plazmoidal state should appear to be stage-like by a continuous increase of the magnetic field. There exists a critical value of the field, which depends on the concentration and atomic weight of the nuclei. A natural frequency of rotation of the plazmoide coincides with the Larmor frequency. By corresponding values of the external magnetic field and the dimensions, the plazmoide should be of a high-temperature type with a temperature on the order of  $10^{10}$  to  $10^{12} \text{ }^\circ\text{K}$ .

The theory establishes quite sufficiently these properties; hence, it is possible to find a similar plazmoide experimentally.

4. Under the conditions of large magnetic fields or sufficiently small concentrations, the found plazmoidal state represents a unique stationary state, characterized by temperature distributions of velocity and a finite circumferential charge per length unit of the cluster.

By a sufficiently large relation of the Langmuir frequency, the Larmor unique stationary state is also violated. There appears a new stationary state which is not already a plazmoidal, but belongs to the class spatial homogeneous propagations. By a sufficiently large relation of the indicated frequencies the plazmoidal state disappears altogether.

5. The integral equation of the state of the plazmoide has the following form:

$$\theta = \frac{m \omega_L^2 D^2}{4} \frac{1}{\psi(\lambda)}$$

where  $\psi(\lambda)$  is expressed by the integral

$$\psi(\lambda) = \int_0^\infty x e^{-x^2 + \phi(x, \lambda)} dx$$

by means of the special function  $\phi(x, \lambda)$ , which is the solution to the following equation:

$$\frac{1}{x} \frac{d}{dx} \left( x \frac{d\phi}{dx} \right) = \lambda e^{-x^2 + \phi}$$

$$\phi(0) = \phi'(0) = 0$$

$$\lambda = 2 \frac{\omega_0^2}{\omega_L^2}; \quad \omega_0^2 = \frac{4\pi e^2 \rho_0}{m}; \quad \omega_L^2 = \left( \frac{eH}{2mc} \right)^2$$

Here will be introduced the results of a numerical integration of the function  $\psi(\lambda)$  which was obtained with the help of the computer "Strela":

| $\lambda$ | $\psi(\lambda)$ |
|-----------|-----------------|
| 0         | 0.5000          |
| $10^{-3}$ | 0.5001          |
| $10^{-2}$ | 0.5009          |
| $10^{-1}$ | 0.5089          |
| 1         | 0.6080          |
| 2         | 0.7903          |
| 3         | 1.1933          |
| 4         | 60.1981         |
| 5         | $> 10^{10}$     |

It is obvious from this table that the condition of the plazmoidal state

$$\int_0^{\infty} x e^{-x^2 + \phi(x, \lambda)} dx < \infty$$

is satisfied by the values  $\lambda < 5$ . In the case of the values  $\lambda$  within the limits from 0 to 1,  $\psi(\lambda)$  changes very little:

$$\frac{1}{2} \geq \psi(\lambda) \geq 0.6080$$

In conclusion, the author would like to express his gratitude to Professors Van Chu-Si and Chao Kay-Chua for the interest shown toward his work and for their special hospitality at the faculty of Physics of the Pekin University.

## APPENDIX I

The necessary properties of coherence of the function  $u_n$  (see Eq. 3.11) which determined the inequality (3.12) will be defined more precisely.

In the mentioned mathematical papers, these properties are connected with a somewhat different equation than Equation (3.10), and either with a finite area of integration, or with special properties of the Grin function and the function  $p(x, y) > 0$  in the equation  $\Delta u - pu = 0$ . In our problem the area of integration is endless and the function  $p(x, y)$  is strictly determined.

1. All  $u_n > 0$ . In fact since the Grin function is always positive (the properties of elliptic equations), then the formula (3.11) indicates that  $u_1 < 0$ . On the other hand if  $u_{n-1} < 0$ , then  $e^{u_n-1} - u_{n-1} > 0$ ; then again from (3.11) one obtains in the case of all  $n$ :  $u_n < 0$ .

2. In the case of arbitrary  $n$ :  $u_n - u_{n+1} > 0$ . Here  $u_0 - u_1 > 0$ , further:

$$u_{n+1} - u_n = -\lambda \iint_{(\infty)} K e^{-(\xi+\eta)+w(\xi,\eta)} [u_{n-1} - u_n + e^{u_n-1} (e^{u_n-u_{n-1}} - 1)] d\xi d\eta \quad (I.1)$$

Assuming that  $u_{n-1} - u_n = x > 0$ , then from the stipulation,

$$1 - e^{-x} < x$$

it follows that

$$u_{n-1} - u_n + e^{u_n-1} (e^{u_n-u_{n-1}} - 1) > (u_{n-1} - u_n)(1 - e^{u_n-1})$$

But on the basis of the first property  $u_{n-1} < 0$  and according to the stipulation  $u_{n-1} - u_n > 0$ , one obtains

$$u_{n-1} - u_n + e^{u_n-1} - 1 (e^{u_n-u_{n-1}} - 1) > 0$$

By utilizing this, one finds from (I.1):

$$u_{n+1} - u_n < 0$$

which was required to be proven.

3. The following inequality takes place

$$u_n - u_{n+1} < \lambda \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} (u_{n-1} - u_n) d\xi d\eta$$

In fact, on the basis of the second property  $e^{u_n - u_{n-1}} < 1$ ; therefore it follows from (I.1) that

$$u_{n+1} - u_n = -\lambda \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} [u_{n-1} - u_n - e^{u_n - u_{n-1}} |e^{u_n - u_{n-1}} - 1|] d\xi d\eta$$

from which one draws the conclusion about the original inequality.

4. The maximum value of  $u_n - u_{n+1}$  will be defined by

$$M_n = \max (u_n - u_{n+1})$$

then on the basis of the third property,

$$M_n \leq M_{n-1} \cdot S_0$$

where

$$S_0 = \max \lambda \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} d\xi d\eta \quad (\text{I.2})$$

Thus, the combined second and third properties might be recorded as follows:

$$0 < u_n - u_{n+1} \leq M_0 S_0^n$$

Also in the preceding, advantage has been taken of this inequality. As is obvious, this inequality preserves its force in spite of dependence of the integration region, on the volume; A special form of the function  $p(x, y) = \exp \{ - (\xi^2 + \eta^2) + w(\xi, \eta) \}$  only increases the similarity of the integral (I.2).

During the proof it has been proposed that in the formula (3.11):

$$\iint_{(\infty)} K \cdot 4\pi\rho \cdot d\xi d\eta < \lambda \iint_{(\infty)} K \cdot e^{-(\xi^2 + \eta^2) + w(\xi, \eta)} (e^{u_n} - u_n) d\xi d\eta$$

Utilizing the determination of  $\rho$ , one will find:

$$\iint_{(\infty)} K \cdot 4\pi\rho d\xi d\eta = \iint_{(\infty)} \lambda e^{-(\xi^2 + \eta^2) + w(\xi, \eta) + u(\xi, \eta)} K \cdot d\xi d\eta$$

since according to the proven  $u_n < 0$  and  $u_{n+1} < u_n$  (consequently, also  $u < u_n$ ), one is convinced that the right part of the last equation is smaller than the right part of the original inequality. Consequently, the original proposition is correct.

## APPENDIX II

The singularity of the solution of the first boundary problem of the equation

$$\Delta \phi = \lambda e^{-x^2 + \phi}$$

when  $\lambda$  is sufficiently small follows from the general structure of the theory on the branching of nonlinear integral equations, which are natural for the equation of Hammershtain (Lichtenstein, Ref. 4). The difference of our equation from those equations examined by mathematicians is confined to the infinite area of integration. However, this is not a principal difference due to the appearance of the "screening" multiplier  $e^{-(\xi^2 + \eta^2)}$ , which can be found in the form of a factor in the Grin function. That effect was proved in the preceding by the example of existence of a solution (Appendix I). An analogous situation can be found in the theorem on the unity of solutions.

Assuming that  $\phi_\lambda$  is the solution with a sufficiently small  $\lambda$ , then one will determine the solution for  $\lambda + \epsilon (\epsilon > 0)$ . Given

$$\Delta \phi_{\lambda + \epsilon} = (\lambda + \epsilon) e^{-x^2 + \phi_{\lambda + \epsilon}}$$

$$\Delta \phi_\lambda = \lambda e^{-x^2} + \phi_\lambda$$

By introducing  $u = \phi_{\lambda + 1} - \phi_\lambda$ , one will obtain

$$\Delta u - \lambda e^{-x^2} u = \epsilon e^{-x^2 + \phi_\lambda} \left( 1 + u + \frac{u^2}{2!} + \dots \right) + \lambda e^{-x^2 + \phi_\lambda} \left( \frac{u^2}{2!} + \frac{u^2}{3!} + \dots \right)$$

In the infinite area this equation equals the following integral equation:

$$u + \frac{\lambda}{2\pi} \iint_{(\infty)} K(x, y; \xi, \eta) e^{-(\xi^2 + \eta^2) + \phi_\lambda(\xi, \eta)} u(\xi, \eta) d\xi d\eta = \phi\{u\} \quad (\text{II.1})$$

where  $\phi\{u\}$  is the nonlinear functional of  $u$  of the following form:

$$\begin{aligned} \phi\{u\} = & -\frac{\epsilon}{2\pi} \iint_{(\infty)} K e^{-(\xi^2 + \eta^2) + \phi\lambda(\xi, \eta)} \left(1 + u + \frac{u^2}{2!} + \dots\right) d\xi d\eta \\ & - \frac{\lambda}{2\pi} \iint_{(\infty)} K e^{-(\xi^2 + \eta^2)} \left(\frac{u^2}{2!} + \frac{u^3}{3!} + \dots\right) d\xi d\eta \end{aligned} \quad (\text{II.2})$$

where the values  $u, u^2$ , etc. are found at the point  $(\xi, \eta)$ . Here  $K(x, y; \xi, \eta)$  is the Green function of the equation  $\Delta u - \lambda e^{-x^2} u = 0$ . The equation (II.1) - (II.2) has a form, which can be found in the basis of the theory of holomorphic and nonholomorphic solutions of nonlinear integral equations. The criterion of the appearance of nonholomorphic (branching) solutions is the presence of a trivial ( $u \equiv 0$ ) solution in the linear equation

$$u + \lambda \iint_{T \rightarrow \infty} K e^{-(\xi^2 + \eta^2) + \phi\lambda(\xi, \eta)} u(\xi, \eta) d\xi d\eta = 0 \quad (\text{II.3})$$

but since this equation is of an elliptical type, it does not have any "natural oscillations" which are necessary for the appearance of branching. This is also indirectly obvious from the differential equation

$$\Delta u - \lambda e^{-x^2 + \phi\lambda} u = 0$$

because

$$\lambda e^{x^2 + \phi\lambda} > 0$$

Therefore the equation (II 1, 2) has a unitary solution, which can be found by the method of analysis by complete (not fractions) degrees of  $\epsilon$ :

$$u = \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \dots$$

From a mathematical viewpoint the initial equation (3.5) is interesting because it has a branching at  $\lambda = 4$ , in spite of the fact that the criterion of the absence of branching is accomplished. This can be explained by the fact that the solution  $\phi_{\lambda=4} = (x^2 + y^2)$  does not belong to the class of functions for which the criterion appears to be actual. The function  $\phi_{\lambda=4}$  cannot be integrated in the investigated endless area:

$$\iint_{(\infty)} \phi_{\lambda=4} d\xi d\eta = \infty$$

## APPENDIX III

Here will be determined the charge propagation and the potential in the case of a cylindrical symmetry of the problem and in the absence of an external magnetic field. From Equation (3.1) one obtains:

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} = \lambda e^\phi \quad (\text{III.1})$$

$$\lambda = \frac{4\pi e^2 \rho_0}{\theta}, \quad \phi = -\frac{eV}{\theta}$$

This equation differs from the equation which was solved by Lemke (Ref. 4) only by the sign and the coefficient  $\lambda$ . The change of the sign, however, is essential for the result. Therefore the obtained solution will be verified.

By introducing the new variables  $x = r^2 e^\phi$  and  $y = r(d\phi/dr)$ , one will find, by determining  $dx$  and  $dy$ , that:

$$\frac{dy}{dx} = \frac{\lambda r e^\phi dr}{(2 + y) r e^\phi dr} = \frac{\lambda}{2 + y}$$

by integration one obtains

$$y^2 + 4y + 2c = 2\lambda x \quad (\text{III.2})$$

where  $c$  is the arbitrary constant of integration, and thus a return to the function

$$\phi \left( y = r \frac{d\phi}{dr} \right)$$

$$r^2 \left( \frac{d\phi}{dr} \right)^2 + 4r \frac{d\phi}{dr} + 2c = 2\lambda r^2 e^\phi \quad (\text{III.3})$$

But on the other hand, the correlation (III.1) has the following form:

$$2 r^2 \frac{d^2 \phi}{dr^2} + 2 r \frac{d\phi}{dr} = 2 \lambda r^2 e^{\phi} \quad (\text{III.4})$$

By comparing (III.3) and (III.4), one obtains:

$$\frac{d^2 \phi}{dr^2} - \frac{1}{r} \frac{d\phi}{dr} - \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 - \frac{c}{r^2} = 0 \quad (\text{III.5})$$

This equation already does not contain an exponential nonlinearity and it is similar to the Riccati equation. It is convenient to introduce the following substitution

$$\phi = -2 \ln w + \ln \tau \quad (\text{III.6})$$

where  $\tau$  is the constant which should be determined; thus one obtains the linear equation for the function  $w$ :

$$\frac{d^2 w}{dr^2} - \frac{1}{r} \frac{dw}{dr} + \frac{c}{2r^2} w = 0 \quad (\text{III.7})$$

The common integral is

$$w = \gamma_1 r^{\beta_1} + \gamma_2 r^{\beta_2}$$

The substitution fits a stipulation on the values  $\beta_1$  and  $\beta_2$ . These values should be the roots of the equation:

$$\beta^2 - 2\beta + \frac{1}{2} c = 0 \quad (\text{III.8})$$

$\gamma_1$  and  $\gamma_2$  are the arbitrary constants. Thus,

$$\phi = -2 \ln (\gamma_1 r^{\beta_1} + \gamma_2 r^{\beta_2}) + \ln \tau$$

Now if one should substitute this function in the original equation (III.1), then he will determine  $\tau$ :

$$\tau = \frac{4 \gamma_1 \gamma_2 (2 - c)}{\lambda}$$

It is not difficult to see that the value of the constant  $c \geq 2$  should be excluded from the investigation, since it leads either to a complexity of  $\phi$  or to a singularity when  $r = 0$ .

In the case of  $c < 2$ ,  $\gamma_1$  and  $\gamma_2$  should have different signs so that  $\ln \tau$  will be the actual value. Thus

$$\phi = -2 \ln \left( \sqrt{\frac{\gamma_1}{\gamma_2}} r^{\beta_1} - \sqrt{\frac{\gamma_2}{\gamma_1}} r^{\beta_2} \right) + \ln \frac{4(2 - c)}{\lambda}$$

where there are two arbitrary constants:  $\gamma_2/\gamma_1$  and  $c$ . In order to satisfy the condition of symmetry on the axis of the cylinder, the intensity of the electrical field should be absent:  $\phi'(0) = 0$ . It is necessary to assume that  $c = 0$ , then from (III.8),  $\beta_1 = 0$ , and  $\beta_2 = 2$ :

$$\phi = -2 \ln \left( \sqrt{\frac{\gamma_1}{\gamma_2}} - \sqrt{\frac{\gamma_2}{\gamma_1}} r^2 \right) + \ln \frac{8}{\lambda}$$

In that manner the solution became singular. It is necessary to connect the singularity with the presence of panels, since otherwise a stationary distribution of similarly charged particles is impossible. There are two possibilities - the radius of the cylindrical panel is arbitrary and precisely determined by the parameters of the system (by the value  $\lambda$ ):

$$\phi = -2 \ln \left[ 1 - \left( \frac{r}{D} \right)^2 \right] + \ln \frac{8}{\lambda D^2}$$

and

$$\phi = -2 \ln \left( 1 - \frac{\lambda}{8} r^2 \right) \quad (\text{III.9})$$

Both cases formally differ only by the normalization of the potential when  $r = 0$ ; they coincide when  $\lambda D^2 = 8$ .

For density distribution, one has

$$\rho = \rho_0 \frac{1}{\left[ 1 - \left( \frac{r}{D} \right)^2 \right]^2}$$

which is the indicated solution quoted from the end of Section III. The apparent singularity in the solution can be eliminated by the introduction of short-lasting refraction forces in the walls, thanks to the activity of which it is also possible to imagine the presentation of reciprocal refraction of particles in the investigated combination.

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